Image Reconstruction in Optical/IR Aperture Synthesis

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Preamble

1/ The problem of optical/IR aperture synthesis imaging is quite different from radio-astronomy:

one cannot rebuild the Fourier phase and produce synthetic complex visibilities (unless perhaps for redundant configuration in snapshot mode, i.e. no hyper-synthesis)

► fit phase closures and power spectrum data

2/ One has to *regularize* in order to:

■ cope with missing data (i.e. interpolate between sampled spatial frequencies)

■ avoid artifacts due to the sparse/non-even sampling

► result is biased toward a priori enforced by regularization it; is important to realize that in order to correctly understand the restored images ► formation of users

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Approximations

- versatile brightness distribution model (no need for FFT's nor rebinning of the sampled spatial frequencies)
- simple model of the data:
  - point-like telescopes (OK as far as $D \ll B$)
  - calibrated powerspectrum and phase closure
- gaussian noise (not true for interferometric data at least because of the calibration)
- probably others ...

Brightness Distribution Model

general linear model of the brightness distribution:

$$z(\mathbf{x}) = \sum_n p_n\, f_n(\mathbf{x}) \quad \xrightarrow{\mathrm{FT}} \quad \hat{z}(\mathbf{u}) = \sum_n p_n\, \hat{f}_n(\mathbf{u})$$

or, using a grid:

$$z(\mathbf{x}) = \sum_n p_n\, f(\mathbf{x} - \mathbf{x}_n) \quad \xrightarrow{\mathrm{FT}} \quad \hat{z}(\mathbf{u}) = \hat{f}(\mathbf{u}) \sum_n p_n\, \mathrm{e}^{-\mathrm{i}\, 2\, \pi\, \mathbf{x}_n\, \mathbf{u}}$$

model of *j*-th complex visibility:

$$\hat{z}(\mathbf{u}_j) = \sum_n a_{j,n}\, p_n$$

with: $a_{j,n} = \hat{f}_n(\mathbf{u}_j)$ or $a_{j,n} = \hat{f}(\mathbf{u}_j)\, \mathrm{e}^{-\mathrm{i}\, 2\, \pi\, \mathbf{x}_n\, \mathbf{u}_j}$

advantages:
- exact Fourier transform
- choice of proper basis of functions (e.g. wavelets, delta functions for stars and splines for background, ...)

Inverse Problem

the image restoration problem is stated as a constrained optimization problem:

$$\mathbf{p}_\mu = \arg\min_{\mathbf{p}} \Psi_\mu(\mathbf{p}|\mathbf{d}) \quad \text{subject to} \quad z(\mathbf{x}) \geq 0, \forall \mathbf{x}$$

penalty: $$\Psi_\mu(\mathbf{p}|\mathbf{d}) = \Psi_{\mathrm{L}}(\mathbf{p}|\mathbf{d}) + \mu\,\Psi_{\mathrm{R}}(\mathbf{p})$$

regularization

hyperparameter

likelihood: $$\Psi_{\mathrm{L}}(\mathbf{p}|\mathbf{d}) = \chi^2_{\mathrm{ps}}(\mathbf{p}|\mathbf{d}_{\mathrm{ps}}) + \chi^2_{\mathrm{cl}}(\mathbf{p}|\mathbf{d}_{\mathrm{cl}})$$

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Likelihood Terms

likelihood for heterogeneous data: $\Psi_{\mathrm{L}}(\mathbf{p}|\mathbf{d}) = \chi^2_{\mathrm{ps}}(\mathbf{p}|\mathbf{d}_{\mathrm{ps}}) + \chi^2_{\mathrm{cl}}(\mathbf{p}|\mathbf{d}_{\mathrm{cl}})$

powerspectrum data: $\chi^2_{\mathrm{ps}}(\mathbf{p}|\mathbf{d}_{\mathrm{ps}}) = \mathbf{r}^{\mathrm{t}}_{\mathrm{ps}} \cdot \mathbf{C}^{-1}_{\mathrm{ps}} \cdot \mathbf{r}_{\mathrm{ps}}$

with residuals: $r_{\mathrm{ps},j} = d_{\mathrm{ps},j} - |\hat{z}(\mathbf{u}_j)|^2$

phase closure data: $\chi^2_{\mathrm{cl}}(\mathbf{p}|\mathbf{d}_{\mathrm{cl}}) = \mathbf{r}^{\mathrm{t}}_{\mathrm{cl}} \cdot \mathbf{C}^{-1}_{\mathrm{cl}} \cdot \mathbf{r}_{\mathrm{cl}}$

with residuals: $r_{\mathrm{cl},k} = \left[d_{\mathrm{cl},k} - \phi(\mathbf{u}_{j_1(k)}) - \phi(\mathbf{u}_{j_2(k)}) + \phi(\mathbf{u}_{j_3(k)})\right]_{\pm\pi}$

$\phi(\mathbf{u}) \equiv \arg[\hat{z}(\mathbf{u})]$ is the Fourier phase

$[\cdot]_{\pm\pi}$ is the difference wrapped in [-π,+π] to avoid the phase wrapping problem (Haniff, 1994)

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Regularization Term

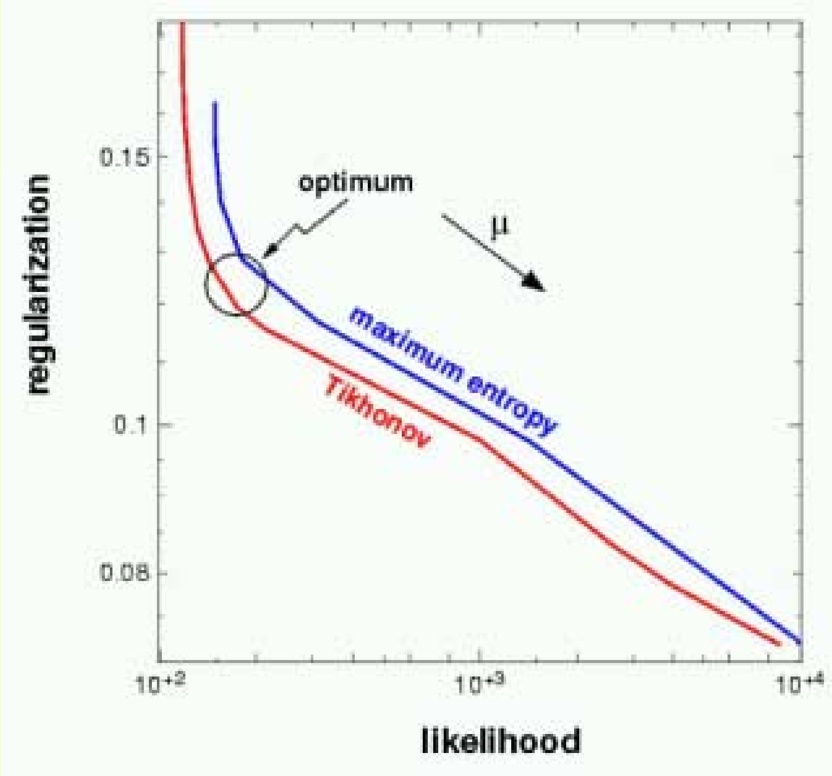
Several possible expressions for the regularization:

- maximum entropy method: $$\Psi_{\mathrm{MEM}}(\mathbf{p}) = \sum_n \left(g_n - p_n + p_n \log \frac{p_n}{g_n} \right)$$
- Tikhonov: $$\Psi_{\mathrm{Tikhonov}}(\mathbf{p}) = (\mathbf{p} - \mathbf{g})^{\mathrm{t}} \cdot \mathbf{R} \cdot (\mathbf{p} - \mathbf{g})$$

  where **g** is the prior, **R** is a symetric positive matrix
- others: ...

Choosing the Hyperparameter(s)

- deterministics methods (e.g. Lannes, Wiener)
- statistics methods, e.g. Gull: $\Psi_\mu(\mathbf{p}_\mu) = \Psi_\mathrm{L}(\mathbf{p}_\mu|\mathbf{d}) + \mu\,\Psi_\mathrm{R}(\mathbf{p}_\mu) = \mathrm{E}\{\Psi_\mathrm{L}(\mathbf{p}_\mu|\mathbf{d})\}$
- cross validation (CV)
- generalized cross validation (GCV, Wahba)
- L-curves (Hansen)



Potential Difficulties

- *heterogeneous data ► more hyperparameters?*
- *possibly large number of parameters*
- *penalty to minimize is:*
  - *non-quadratic ► non-linear optimization*
  - *multi-mode (sum of terms with different behaviour)*
  - *constrained (at least positivity)*
  - *non-convex ► multiple local minima*

  □ *very difficult to optimize*
- *phase wrapping problem (solved)*

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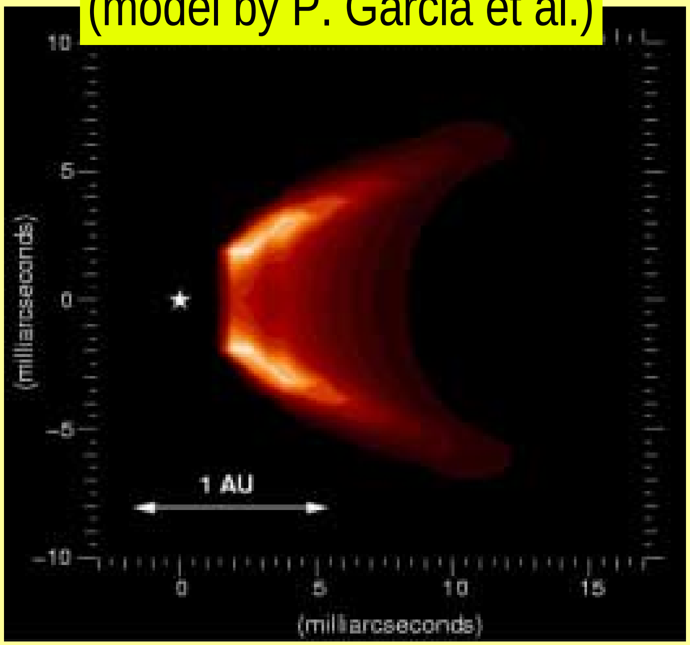
Optimization Part

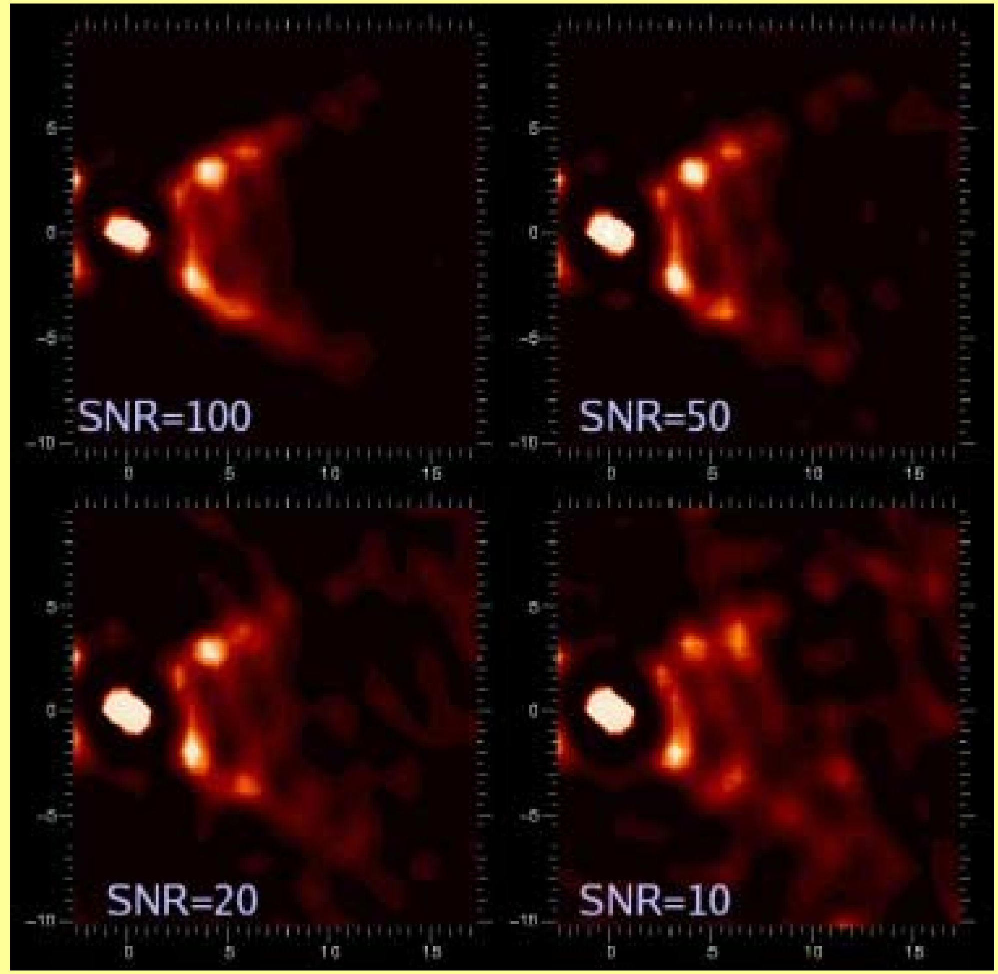
optimization of a non-convex, non-quadratic penalty function of a large number of constrained parameters by:

- descent methods:
  - *variable metric* methods (BFGS) are faster than conjugate gradient
    - there exists limited memory version (VMLM, Nodedal 1980)
    - can be modified to account for bound constraints (VMLM-B, Thiébaut 2002)
    - easy to use (only gradients required)
  - *local subspace* method should be more efficient (Skilling & Brian 1984; Thiébaut 2002) but needs second derivatives
- global methods?

Test Image: PMS's Microjet

microjet emitted by a PMS
(model by P. Garcia et al.)





- 7 observing nights
- 7 configurations with 3 AT's
  - 190 powerspectrum data
    - 63 phase closures
- 1024 unknowns

Future Work for the Image Restoration Software

- account for correlated data (+)
- use data exchange format (+)
- automatically adjust hyperparameters (++)
- improve optimization part (+++++)
- link with ASPRO (G. Duvert) for more realistic simulated data
- provide *error bars* (++)
- *process real data (Amber with 3 telescopes in 2004)*

Future Work for the Image Restoration Group of the JMMC

- elaborate on *proper regularization(s)*
- *model of the data may be more complex*
- metric to compare restored images with different
  - configurations → optimization of (u,v) coverage to *reduce observing time*
  - regularizations
- estimation of the best hyperparameters
- educate astronomers (summer school, workshops, ...): *regularized image reconstruction is not so difficult to understand and must be understood to realize the unavoidable biases in the result*